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# Grading Rubric

**All work in this workbook needs to be in pencil.**

	<b>0-1 points</b>	<b>2-3 points</b>	<b>4-5 points</b>
Completeness	No parts have been completed. Majority of definitions and example work is missing.	Some information is written down. Some steps or parts are missing. Ordered pairs or scale from the graphs are missing.	All definitions and examples are completed, no steps missing
Neatness	The information is not clear or comprehensible.	There are some parts that are not clear. Some steps are missing or not clear.	All work is neatly written and clear. A final answer is circled.
Organization	Information is not presented in the right or designated place.	Some information is not presented in the designated place or is presented inappropriately. Three or fewer steps don't follow logically. Some equal signs are missing.	All information is in the right and/or designated place. There is a logic flow to all work. No equal or mathematical signs are missing.
Correctness	There are many mistakes in the work or the definitions.	Three or fewer mistakes in the definitions or worked examples.	No mistakes are made and all work is correct.
			<b>TOTAL 20 Points</b>

## Class 1 – Review

**Set:** \_\_\_\_\_

The following are examples of sets

- **Natural numbers:** \_\_\_\_\_
- **Whole numbers:** \_\_\_\_\_
- **Integers:** \_\_\_\_\_
- **Rational numbers** \_\_\_\_\_
- **Irrational numbers**
- **Real numbers**
- **Prime number**

Practice: Give at least three examples of prime numbers \_\_\_\_\_

- **Absolute Value** of a number  $a$ , denoted by  $|a|$ , is \_\_\_\_\_

*Example:*

1.  $|4| =$

2.  $|-3| =$

## Fractions

A Fraction is \_\_\_\_\_

Give a few examples \_\_\_\_\_

- **Simplifying**: To simplify a fraction we

$$\text{Simplify: } \frac{32}{24} =$$

- **Reciprocal** of a fraction is \_\_\_\_\_

$$\text{Find the reciprocal of } \frac{3}{7}$$

- **Multiplying**: To multiply two fractions we \_\_\_\_\_

$$\text{Multiply: } \frac{11}{2} \cdot \frac{4}{3} =$$

- **Dividing**: To divide two fractions we \_\_\_\_\_

$$\text{Divide: } \frac{11}{3} \div \frac{5}{6}$$

- **Adding/Subtracting**: To add/Subtract fractions we \_\_\_\_\_

- **LCD is** \_\_\_\_\_

Add:  $\frac{11}{3} + \frac{5}{2}$

Practice on the following problems:

3.  $\frac{5}{11} + \frac{7}{11} =$

4.  $\frac{3}{4} - \frac{5}{7} =$

5.  $\frac{5}{12} + \frac{3}{20} =$

6.  $\frac{12}{28} + \frac{5}{24} =$

## Exponential Expression

An **exponential expression** is an expression of the form \_\_\_\_\_

Give at least two examples: \_\_\_\_\_

Simplify

$$7. 7^1 =$$

$$8. \left(\frac{2}{3}\right)^3 =$$

$$9. (-3)^4 =$$

$$10. -5^2 =$$

### Exponential Rules:

Write the exponential rules for each of the following:

$$\bullet a^x \cdot a^y = \underline{\hspace{2cm}}$$

$$(ab)^x = \underline{\hspace{2cm}}$$

$$a^1 = \underline{\hspace{2cm}}$$

$$\bullet \frac{a^x}{a^y} = \underline{\hspace{2cm}}$$

$$\left(\frac{a}{b}\right)^x = \underline{\hspace{2cm}}$$

$$a^0 = \underline{\hspace{2cm}}$$

$$\bullet (a^x)^y = \underline{\hspace{2cm}}$$

$$a^{-x} = \underline{\hspace{2cm}}$$

$$a^{x/y} = \underline{\hspace{2cm}}$$

Practice: Simplify the following:

$$11. \left(\frac{r}{s}\right)^6 =$$

$$12. \left(\frac{5x^6}{9y^3}\right)^2 =$$

$$13. \left(\frac{9x^3}{y}\right)^{-2} =$$

$$14. \frac{6^{-5}x^{-1}y^2}{6^{-2}x^{-4}y^4} =$$

$$15. \frac{(3x^{-2}y)^{-2}}{4x^7y} =$$



## Order of Operation ( P-E-M-D-A-S )

- 
- 
- 
- 

Example: Evaluate the following.

$$16. \frac{8 \cdot 2^3 - |3 - 7| + 5(3^2 - 4)}{5 \cdot 2 - 7} =$$

- **Scientific Notation:**

A number is written in scientific notation if \_\_\_\_\_

Give an Example: \_\_\_\_\_

17. Practice: Write the number in Scientific Notation

$$9,060,000,000 =$$

$$0.00017 =$$

18. Write the number in Decimal Notation

$$3.067 \times 10^{-4} =$$

$$6.002 \times 10^6 =$$

19. Perform each operation and write in standard form and Scientific Notation

$$(9 \times 10^7)(4 \times 10^{-9})$$

$$\frac{8 \times 10^4}{2 \times 10^{-3}} =$$

**Radicals:****In General:** \_\_\_\_\_**Notation** \_\_\_\_\_

Examples:  $\sqrt{100} =$        $-\sqrt{36} =$        $\sqrt{-1} =$        $\sqrt{\frac{25}{81}} =$

- **Product Rule:** \_\_\_\_\_
- **Quotient Rule:** \_\_\_\_\_
- **Simplify:** \_\_\_\_\_

Simplify:  $\sqrt{4x^6} =$        $\sqrt{\frac{16y^{24}}{81}} =$

- **Add/Subtract:** \_\_\_\_\_

Add the following:

$$\sqrt{28} + \sqrt{63} =$$

$$\sqrt{72x^2} + \sqrt[3]{54} - x\sqrt{50} - 3\sqrt[3]{2} =$$

- **Multiply** \_\_\_\_\_

Multiply the following:

$$2\sqrt{3} \cdot 7\sqrt{5} =$$

$$\sqrt[4]{8} \cdot \sqrt[4]{2} =$$

**More Definitions:**

- **Variables** \_\_\_\_\_

Give at least three *examples*: \_\_\_\_\_

- **Algebraic Expression** \_\_\_\_\_

Give a few *examples*: \_\_\_\_\_

We can *Evaluate* algebraic expressions if we know the value of the variable(s).

*Example: Evaluate  $2x - y^2$  if  $x = 3$  and  $y = 2$*

\_\_\_\_\_

- **Equation:** \_\_\_\_\_

Give two *Examples*: \_\_\_\_\_

- **Solution/Root** \_\_\_\_\_

Check to see if  $x = 2$  is a solution to the equation  $5x - 3 = 4x - 1$

## Polynomials

A **Polynomial in  $x$**  is \_\_\_\_\_

Give at least three examples \_\_\_\_\_

### **Fun things we do with polynomials**

- **Evaluating:** \_\_\_\_\_

Find the value of the polynomial  $6x^2 + 11x - 20$  when  $x = -1$ .

- **Simplifying, Adding and Subtracting:** \_\_\_\_\_

*Add/Subtract the following:*

$$14y^2 + 3 - 10y^2 = \underline{\hspace{10em}}$$

$$(5x^2 - 2x + 1) - (-6x^2 + x - 1) = \underline{\hspace{10em}}$$

- **Multiplying:** \_\_\_\_\_

*Multiply:*  $(5y^2 - 6y + 7)(4y + 3) =$

## Special Products

- **FOIL** = \_\_\_\_\_

Foil the following:  $(x+7)(x-5) =$

- **Squaring a Binomial:**  $(a+b)^2 =$  \_\_\_\_\_ or  $(a-b)^2 =$  \_\_\_\_\_

Square the following:  $(2x+5)^2 =$

- **Difference of Squares:**  $(a+b)(a-b) =$  \_\_\_\_\_

*Multiply the following:*

$$\left(x - \frac{1}{3}\right)\left(x + \frac{1}{3}\right) = \underline{\hspace{2cm}}$$

$$(2x^2 + 6x)(2x^2 - 6x) = \underline{\hspace{2cm}}$$

- **Dividing Polynomials**

- Dividing by a **Monomial:** \_\_\_\_\_.

$$\text{Divide: } \frac{25x^3 + 5x^2}{5x} =$$

- Dividing **by a polynomial other than a monomial:** \_\_\_\_\_

$$\text{Divide: } \frac{x^2 + 7x + 12}{x + 3} =$$

## Factoring

- Factoring is the process of \_\_\_\_\_
- GCF of a list of Integers \_\_\_\_\_
- To find the GCF \_\_\_\_\_  
\_\_\_\_\_

Find the GCF(45, 75) \_\_\_\_\_

Find the GCF of the following numbers:

$$32 \text{ and } 33 =$$

$$24, 60 \text{ and } 96 =$$

- GCF of a list of Common Variables \_\_\_\_\_

Example: Find the GCF( $x^5, x^3$ )  $\rightarrow$  \_\_\_\_\_

- GCF of a list of a list of TERMS \_\_\_\_\_

Find the GCF( $-9x^2, 15x^4, 6x$ ):

The first step to factoring a polynomial

- **Prime Polynomial** \_\_\_\_\_
- **Factoring by Grouping is used for** \_\_\_\_\_
- **To Factor by Grouping** \_\_\_\_\_

Factor the polynomial  $ab + 4a + 7b + 28 =$

*Practice: Factor the following polynomials*

21.  $5a^2 + 2ab - 5a - 2b$

22.  $15xz + 15yz - 5xy - 5y^2$

- **Factoring Trinomials of the form**  $x^2 + bx + c$

*Example*  $x^2 + 9x + 20 =$

*Practice: Factor the following*

23.  $x^2 - 13x + 22 =$

24.  $x^2 + 5x - 36 =$

- **Factoring Trinomials of the form**  $ax^2 + bx + c$
- 

*Example*      $2x^2 + 5x - 12$

### Factoring Binomials

- **Difference of two squares:**  $a^2 - b^2 =$  \_\_\_\_\_

Factor  $a^2 - 16 =$

Practice on factoring the following:

$25x^2 - 1 =$

$p^4 - 81 =$

$48x^4 - 3 =$

$c^2 - \frac{9}{25} =$

- $a^2 + b^2$  is \_\_\_\_\_

- **Sum/Difference of two Cubes:**  $x^3 + y^3 =$  \_\_\_\_\_      $x^3 - y^3 =$  \_\_\_\_\_

Factor the following:  $125x^3 - 1 =$

$16p^3 + 250y^3 =$



## Rationals

- **Rational Expression** \_\_\_\_\_  
\_\_\_\_\_

- **Domain** \_\_\_\_\_

- **To find the domain** \_\_\_\_\_

Ex:  $\frac{2x-5}{x+3}$  is defined for \_\_\_\_\_

Example: Find the domain of the following expressions:

$$\frac{4}{x-2},$$

$$\frac{x}{x^2-1},$$

$$\frac{2x-3}{6x^2-5x+1}$$

### ➤ **Operations** \_\_\_\_\_

- **Simplifying:** \_\_\_\_\_.

Simplify  $\frac{x^2+6x+5}{x^2-25}$

- **Multiplying:**
- 

Multiply  $\frac{x-7}{x-1} \cdot \frac{x^2-1}{3x-21}$

- **Dividing:**
- 

Divide:  $\frac{x^2-2x-8}{x^2-9} \div \frac{x-4}{x+3}$

- **Adding & Subtracting:**
- 

Add:  $\frac{x+3}{x^2+x-2} + \frac{2}{x^2-1}$

**Complex Fractions**

- To simplify complex fractions \_\_\_\_\_

Simplify:  $\frac{\frac{1}{x} - \frac{3}{2}}{\frac{1}{x} + \frac{3}{4}}$

Simplify:  $\frac{2d}{\frac{d}{r_1} + \frac{d}{r_2}}$

## Class 2: Linear Models

### Objectives:

- ~ Graph Equations on the Rectangular coordinate system.
- ~ Solve Linear Equations in One Variable
- ~ Solve Rational Equations with Variables on the denominator
- ~ Use Linear Equations to Solve Problems.
- ~ Solve a Formula for a Variable

### o The Rectangular Coordinate System comprises of \_\_\_\_\_

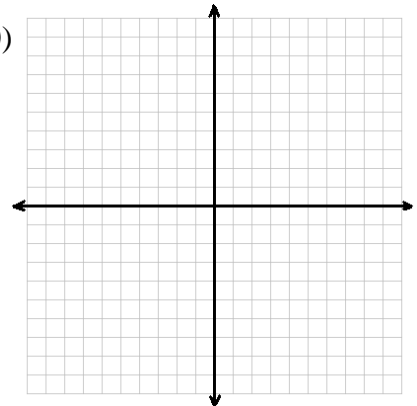
Draw and label a rectangular coordinate system below:

The way we plot a point (a,b) in the coordinate system is \_\_\_\_\_

---

Practice 1: Plot the following points in the coordinate system below.

(4, 2), (2, -2), (-1, -3), (-5, 1), (0, 2), (3, 0), (0, -4), (-4, 0)



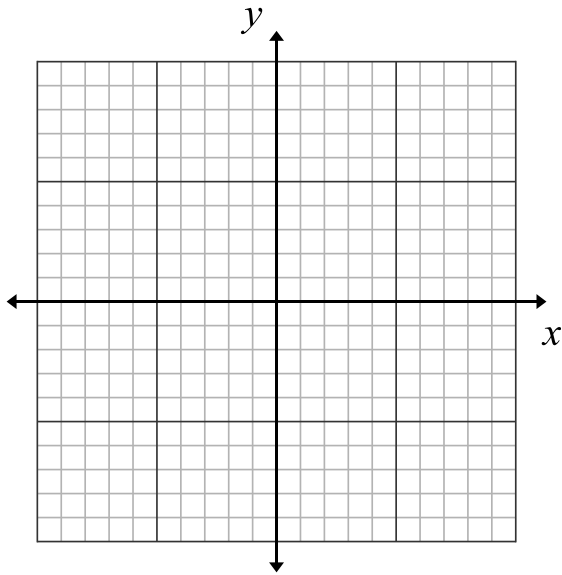
- An equation in two variables such as  $2x - y = 1$  or  $y = 4x^2 + 3$  has a solution \_\_\_\_\_

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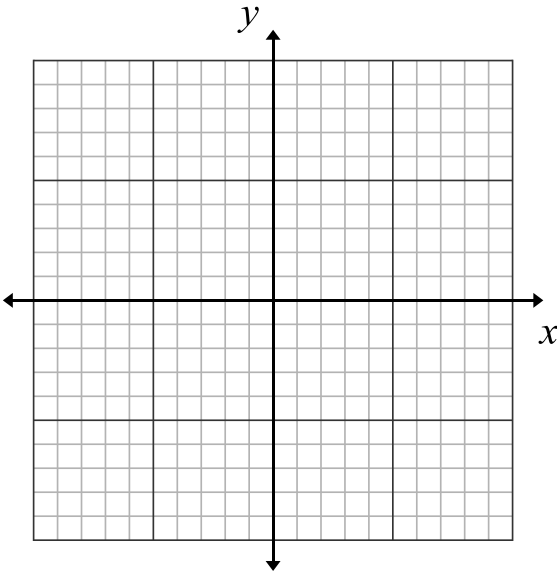
- **Graph of an Equation**

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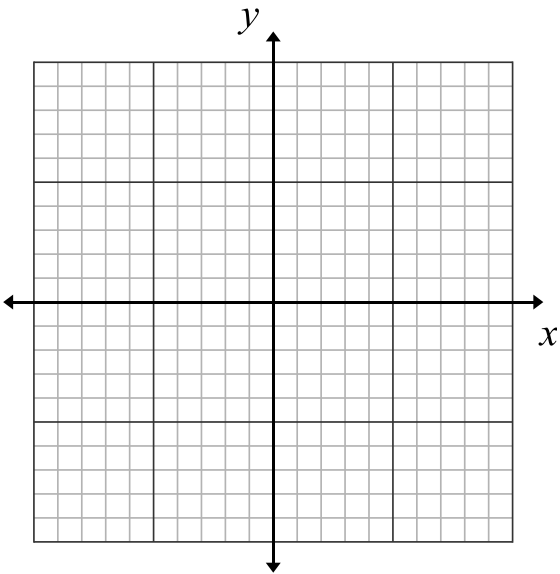
Practice 2: Graph the equation  $y = 2x - 3$  by using the point-plotting method.



Practice 3: Graph the equation  $y = x^2 + 3$  by using the point-plotting method.



Practice 4: Graph the equation  $y = |x + 3| - 2$  by using the point-plotting method.



- **Intercepts**

- **X-Intercept is** \_\_\_\_\_
- **To find the x-intercept** \_\_\_\_\_
- **Y -Intercept is** \_\_\_\_\_
- **To find the x-intercept** \_\_\_\_\_

Practice 5: Find the x and y-intercepts of  $2x - y = 12$

## Solving Equations

### General Strategy of Solving Linear Equations

---

1.

2.

3.

4.

5.

6.

---

Practice: Solve each of the equations

6.  $-6(2x+1)-14 = -10(x+2)-2x$

7.  $\frac{2}{5} - \frac{1}{x} = \frac{3}{4}$

$$8. \frac{5-x}{3} = 2x-7$$

➤ Literal Equations are

---

Solve each of the following for s

$$9. \quad V = C - \frac{C-S}{L}N$$



## Word Problems

<u>General Strategy of Solving Word Problems</u>
1.
2.
3.
4.

*Example:*

1. Twice the difference of a number and 8 is equal to three times the sum of the number and 3. Find the number.

Step 1:

Step 2:

Step 3:

Step 4:

2. To make an international call, you need the code for the country you are calling. The codes for Belgium, France and Spain are three consecutive integers whose sum is 99. Find the code for each country.

Step 1:

Step 2:

Step 3:

Step 4:

Practice:

15. The sum of twice a number and 7 is equal to the sum of a number and 6.
16. If  $\frac{3}{4}$  is added to three times a number, the result is  $\frac{1}{2}$  subtracted from twice the number.
17. The room numbers of two adjacent classrooms are two consecutive even numbers. If their sum is 654, find the classroom numbers
18. A 40-inch board is to be cut into three pieces so that the second piece is twice as long as the first piece and the third piece is 5 times as long as the first piece. Find the lengths of all three pieces.

**Formulas**

$$A = lw$$

---

$$P = 2l + 2w$$

---

$$P = a + b + c$$

---

$$A = \frac{1}{2}bh$$

---

$$V = lwh$$

---

$$A = \pi r^2$$

---

$$P = 2\pi r$$

---

$$d = rt$$

---

$$I = PRT$$

---

$$F = \frac{9}{5}C + 32$$

---

Further Problem Solving

---

- Solve problems involving Percents

- Increase \_\_\_\_\_

- Percent Increase =

- Decrease \_\_\_\_\_

- Percent Decrease =

**21.** Nordstrom's advertised a 25% off sale. If a London Fog coat originally sold for \$256, find the decrease in price and the sale price

**23.** How many cubic centimeters (cc) of a 25% antibiotic solution should be added to 10cc of a 60% antibiotic solution to get a 30% antibiotic solution?

**25.** A jet plane traveling at 500mph overtakes a propeller plane traveling at 200mph that had a 2-hour head start. How far from the starting point are the planes?

**26.** Karen invested some money at 9% annual simple interest and \$250 more than that amount, at 10% annual simple interest. If her total yearly interest was \$101, how much was invested in each?

## Class 3: Quadratics

### Objectives:

- ~ Perform Operations with Complex Numbers
- ~ Solve Quadratic Equations by any method
- ~ Solve Polynomial Equations by factoring
- ~ Solve Radical Equations.
- ~ Solve Equations with Rational Exponents
- ~ Solve Equations involving Absolute Values
- ~ Solve linear and Absolute Value Inequalities

- **The Imaginary Unit is** \_\_\_\_\_
- **A complex Number is** \_\_\_\_\_  
\_\_\_\_\_
- **Complex Conjugate** \_\_\_\_\_

### ➤ Operations with Complex Numbers

- **Powers of Imaginary Numbers.**  
Practice 1. Perform the indicated Operation

$$(i)^5 =$$

$$(-2i)^7 =$$

**Addition/Subtraction:** \_\_\_\_\_

Subtract:  $(-7 + 5i) - (-9 - 11i) =$

**Multiplication:** \_\_\_\_\_

Multiply:  $(9 - 45i)(5 - i)$

○ **Division:** \_\_\_\_\_

Practice 5:  $\frac{8 + 5i}{8 - 5i}$

## Quadratic Equation

- A Quadratic Equation \_\_\_\_\_  
\_\_\_\_\_

- **Zero Factor Theorem:** \_\_\_\_\_

*Example:*  $x^2 - 5x - 14 = 0$

- **Factoring:**

### Solving Quadratic Equations by Factoring:

1.

2.

3.

4.

5.

Solve:  $-5x^2 + 20x + 60 = 0$

- **Square Root Property**

$$\text{If } x^2 = a \text{ for } a \geq 0 \Rightarrow x = \pm\sqrt{a}$$

Example:  $x^2 - 49 = 0 \Rightarrow$

$x =$  \_\_\_\_\_

Solve by the square root property:  $(x - 4)^2 = 36$

$x =$  \_\_\_\_\_



## Completing the Square

- To complete the square \_\_\_\_\_

Example  $x^2 + 8x + 1 = 0$

### General Strategy for Completing the Square

---

- 1.
  - 2.
  - 3.
  - 4.
  - 5.
- 

**Example:**  $x^2 + 2x = 4$

## Quadratic Formula

- The Quadratic Formula \_\_\_\_\_

Solve by using the Quadratic Formula:  $2x^2 - x - 5 = 0$

Practice 6:  $3x^2 + 8x = 3$

- The Discriminant \_\_\_\_\_

Discriminant	Number of Solutions

*Practice:* Use the discriminant to find the # of solutions

7.  $5x^2 + 2x - 3 = 0$

8.  $x^2 + 2x + 2 = 0$

9.  $x^2 + 2x + 1 = 0$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

## Other Equations

- **Radical Equation** \_\_\_\_\_

Give at least two Examples \_\_\_\_\_

- **Domain** \_\_\_\_\_

Find the domain  $\sqrt{x-4} = 9$

\_\_\_\_\_

### Strategy on Solving Radical Equations containing nth Roots

1.

2.

3.

4.

Solve the equation:  $15 + \sqrt{3x+17} = x$

**Equations with Rational Exponents:**

- Equations with rational exponents are \_\_\_\_\_

Rewrite as radical:  $a^{\frac{m}{n}} =$  \_\_\_\_\_

**Strategy on Solving Equations with Rational Exponents**

1.

2.

3.

4.

Solve the following equations for their real solutions.

10.  $8x^{\frac{5}{3}} - 24 = 0$

11.  $2x^{\frac{2}{3}} + 4x^{\frac{1}{3}} = 6$

## Equations Involving Absolute Values

$$|x| = \left\{ \begin{array}{l} \text{_____} \\ \text{_____} \end{array} \right.$$

Solve:  $5|3x-7|+6=21$

## Inequalities

- **Linear Inequalities** \_\_\_\_\_

Solve the following inequality  $5x+11 < 26$

**Absolute Value Inequalities**

If  $u$  is an algebraic expression and  $c$  is a positive number, then

$$|u| < c \quad \underline{\hspace{15em}}$$

And

$$|u| > c \quad \underline{\hspace{15em}}$$

Solve the following inequality

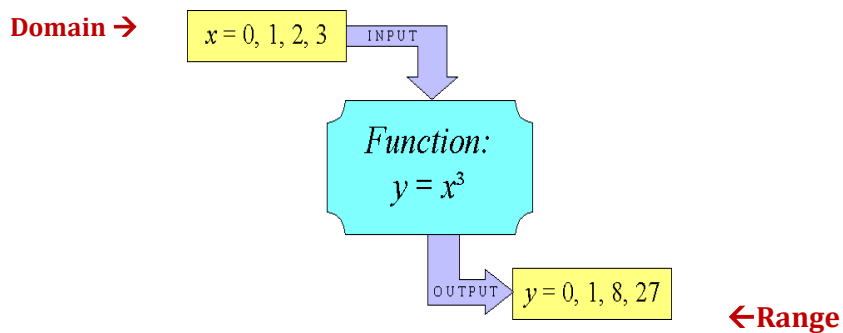
$$7|2x-8|+14 \geq 35$$

## Class 4: Function and Their Graphs

### Objectives:

- ~ Identify and Graph Functions
- ~ Identify Domain and Range
- ~ Identify Characteristics of Functions
- ~ Calculate the slope of a Line.
- ~ Write and find the point-Slope and Slope intercept of the equation of a line
- ~ Solve Equations involving Absolute Values

- **A Relation is** \_\_\_\_\_
- **Domain** \_\_\_\_\_
- **Range** \_\_\_\_\_
- **Functions is** \_\_\_\_\_



**There are four possible ways to represent a function: List them below:**

- 1.
- 2.
- 3.
- 4.

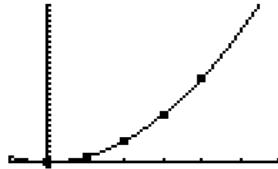
Example:

- **Verbally:** The area of a square plot of land is equal to the square of the length of the lot.
- **Numerically:** (0,0), (1,1), (2,4), (3,9), (4,16) ...

Or

Length	0	1	2	3	4	...
Area	0	1	4	9	16	...

- **Visually:**



- **Algebraically:**  $A(s) = s^2$

- **Notation:**

A function  $f$  of  $x$  is represented as: \_\_\_\_\_

$x$  - represents \_\_\_\_\_

$y$  - represents \_\_\_\_\_

The Graph of a Function \_\_\_\_\_

➤ **Determining whether a relation is a function**

- **Numerically**

**Practice:** Determine if the following examples are functions. If not, explain.

1. In the following ordered pairs the first element represents "Number of hours worked" and the second element represents "Total pay".

(0, \$0)	(1, \$7.50)	(2, \$15.00)
(3, \$22.50)	(4, \$30.00)	(5, \$37.50)
(6, \$45.00)	(7, \$52.50)	(8, \$60.00)

2. The first element of each ordered pair is "Student First Name" and the second element of each ordered pair is "Number of Math Courses Taken".

1. (Peter, 2)	2. (Jackie, 0)	3. (Marian, 2)
4. (Tammy, 3)	5. (Jess, 1)	6. (Jackie, 1)
7. (John, 3)	8. (Joe, 2)	9. (Ron, 0)



○ **Algebraically**

To determine if an equation is a function \_\_\_\_\_  
 \_\_\_\_\_

**Practice:** Determine if the following equations define y as a function of x.

3.  $x^3 + y = 14$

4.  $x^3 + y^2 = 14$

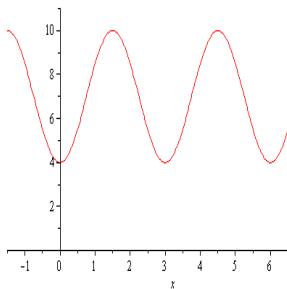
5.  $xy + 3y = 4$

○ **Visually**

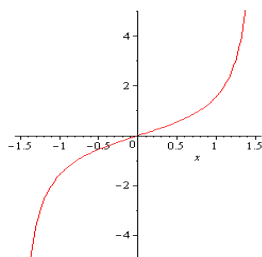
• **The Vertical Line Test :** \_\_\_\_\_  
 \_\_\_\_\_

**Practice:** Determine if y is a function of x.

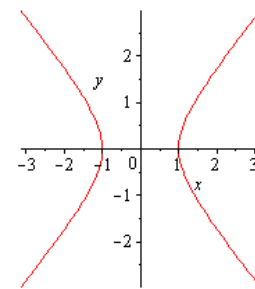
6.



7.



8.



**Graph of a Function**

- Arrows indicate \_\_\_\_\_
- A closed dot • indicates \_\_\_\_\_
- An open dot ◦, indicates \_\_\_\_\_

➤ **Finding Domain and Range**

Practice: Find the Domain and Range in each of the following cases:

○ **Numerically**

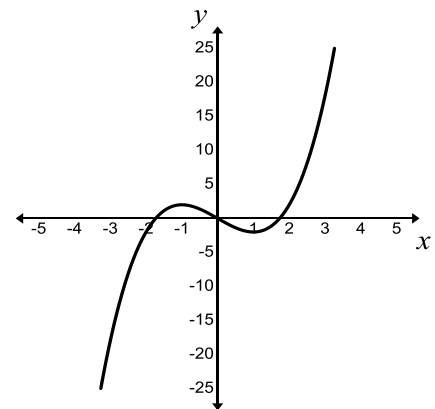
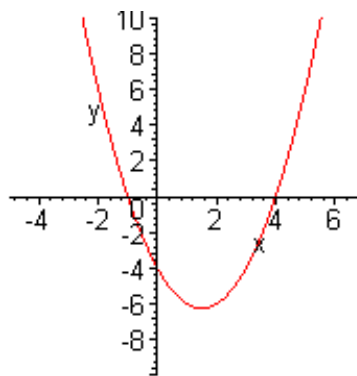
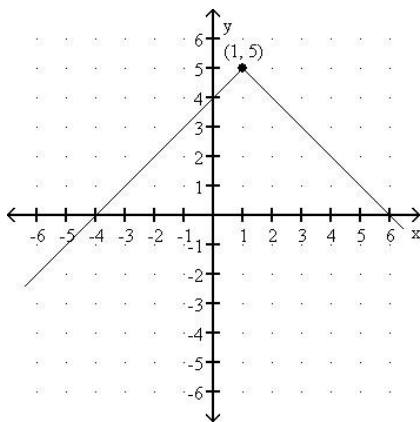
7. (0, 1650), (10, 1750), (20, 1860), (30, 2070), (40, 2300),  
 (50, 2560), (60, 3040), (70, 3710), (80, 4450), (90, 5280)

Domain \_\_\_\_\_

Range \_\_\_\_\_

○ **Visually**

8.



Domain: \_\_\_\_\_

Range: \_\_\_\_\_

[http://lima.osu.edu/people/iboyadzhiev/GeoGebra/domain\\_range.html](http://lima.osu.edu/people/iboyadzhiev/GeoGebra/domain_range.html)

- **Algebraically**

The Domain of any polynomial function is \_\_\_\_\_

**Exceptions:**

- \_\_\_\_\_
- \_\_\_\_\_

Give an example of each of the above exceptions:

\_\_\_\_\_

➤ **Evaluating Functions**

Same process as evaluating an algebraic expression

9. Example: Consider the function  $f(x) = 2x^2 - 5x + 3$  . Evaluate the following:

a.  $f(-3)$

b.  $f(h)$

c.  $f(h+2)$

**Piece-Wise Functions**

10. Practice: Graph the following function.

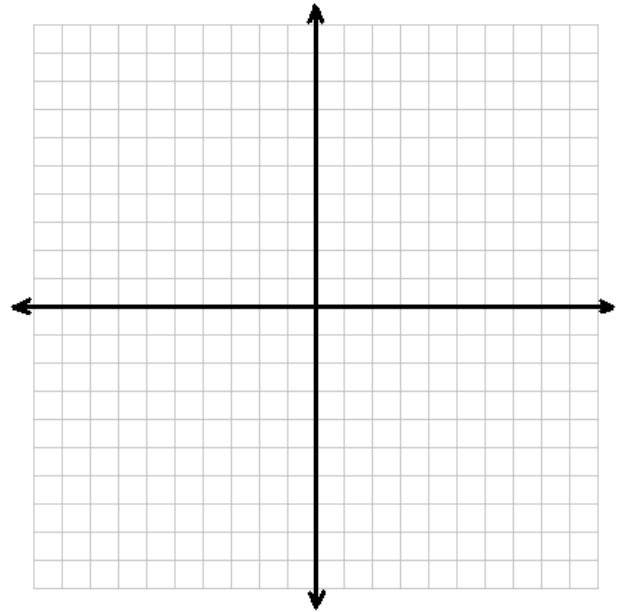
$$f(x) = \begin{cases} x - 3, & \text{if } x < 0 \\ 4, & \text{if } x = 0 \\ x + 6, & \text{if } x > 0 \end{cases}$$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

Evaluate  $f(-2) =$   
 \_\_\_\_\_

$f(3) =$   
 \_\_\_\_\_



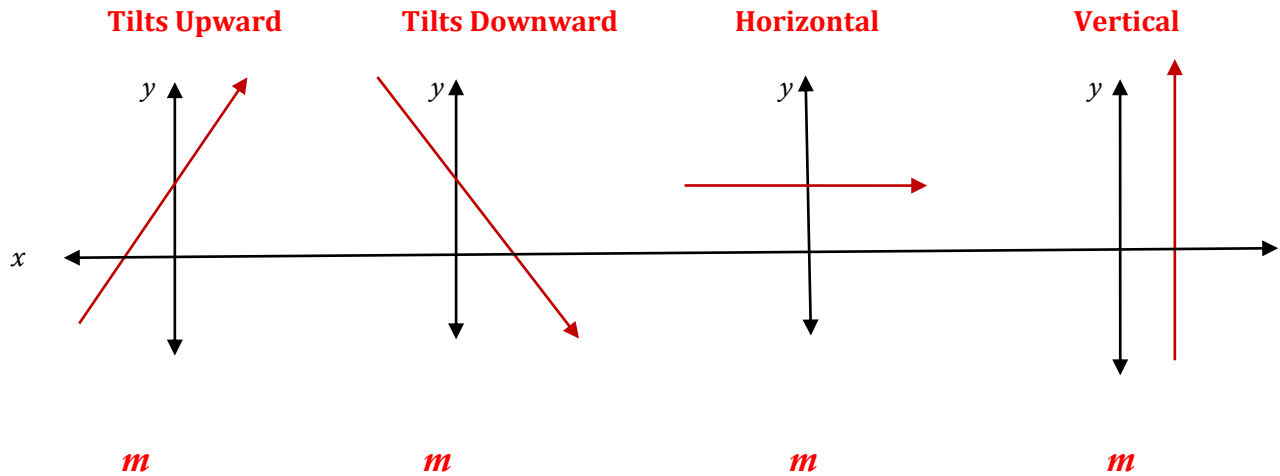
**Characteristics of Functions:**

- **DOMAIN** - \_\_\_\_\_
- **RANGE** - \_\_\_\_\_
- **MAX/MIN** - \_\_\_\_\_
- **Increase/Decrease** - \_\_\_\_\_
- **X-INTERCEPTS** - \_\_\_\_\_
- **Y-INTERCEPTS** - \_\_\_\_\_
- **Odd** - \_\_\_\_\_
- **Even** - \_\_\_\_\_

## Linear Functions and Slope

- Write the General Form of the Equation of a Line: \_\_\_\_\_
- Write the equation of a horizontal line: \_\_\_\_\_
- Write the equation of a vertical line: \_\_\_\_\_
- Slope is \_\_\_\_\_
- Write the formula used to find the slope of a line \_\_\_\_\_
- Write the slope -intercept form of a linear equation and state what each part represent.  
\_\_\_\_\_
- Write the Point-Slope form of the Equation of Line: \_\_\_\_\_
- State the appropriate slope for each of the following cases:  
 Vertical Line \_\_\_\_\_  
 Horizontal Line \_\_\_\_\_

11. Fill in the appropriate slope for each of the lines below:



Practice: For the each of the following find the slope of the line through the points:

12.  $(-2, -5)$ ,  $(0, -2)$ ,  $(4, 4)$ ,  $(10, 13)$

13.  $(-2, 1)$ ,  $(3, 5)$

14. State the slope of each of the lines given by the equations below:

a.  $y = 3x - 5$

b.  $y = -\frac{x}{7} + 4$

m = \_\_\_\_\_

m = \_\_\_\_\_

15. Find the equation of the line that goes through the points  $(-2, 3)$  and  $(-5, -1)$ .

## Class 5: Average Rate of Change and Transformations

### Objectives:

- ~ Calculate Average Rate of Change
- ~ Calculate the Difference Quotient
- ~ Recognize Graphs of Common Functions
- ~ Use transformations to graph Functions

- **Average Rate of Change:** \_\_\_\_\_

Practice: For each of the following functions, find the average rate of change.

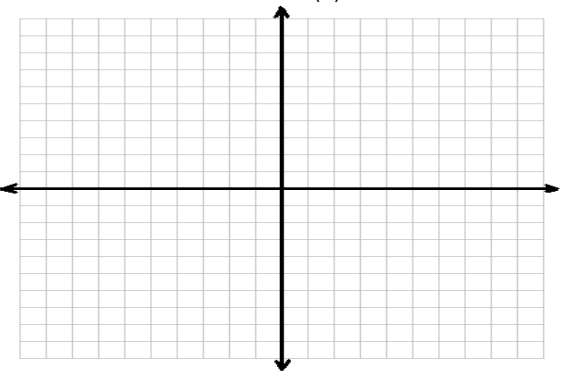
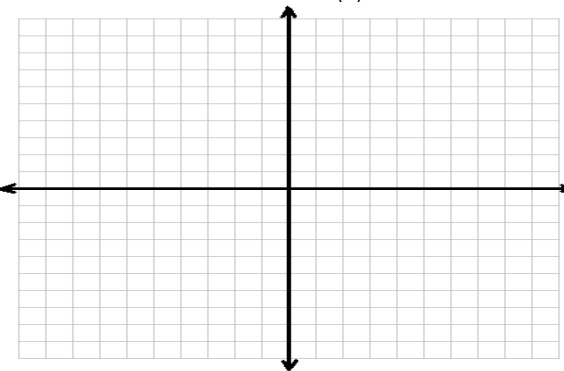
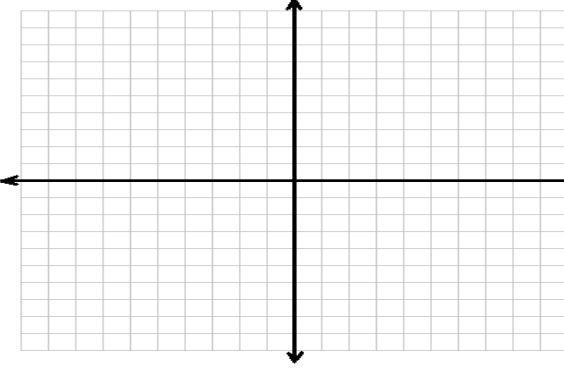
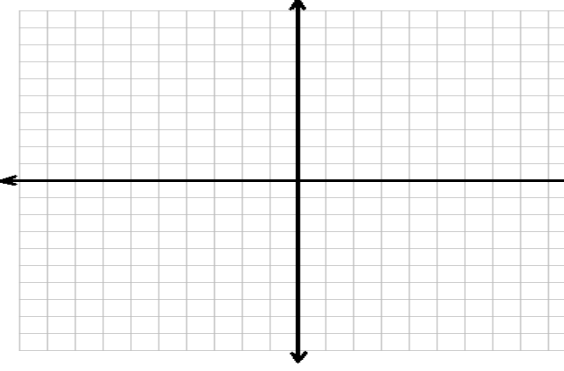
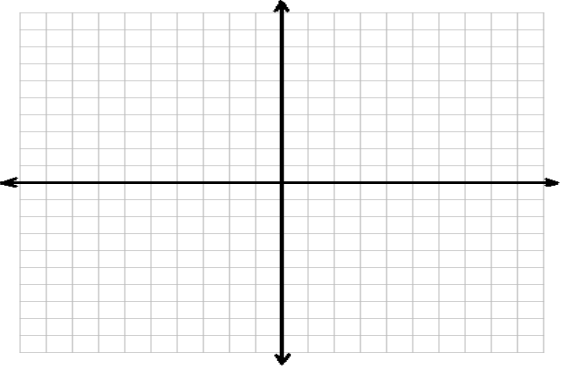
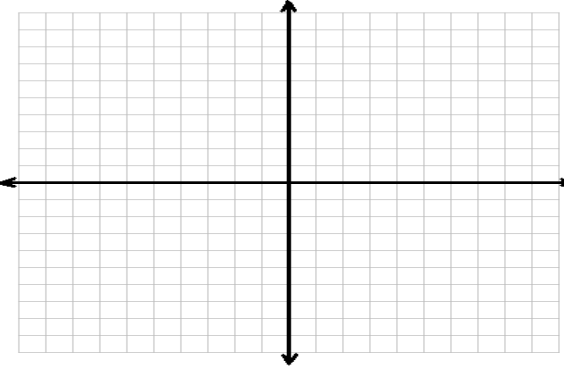
1.  $f(x) = x^2 - 2x + 2$   $x_1 = 3$  to  $x_2 = 6$

### Difference Quotient

- **Difference Quotient** \_\_\_\_\_

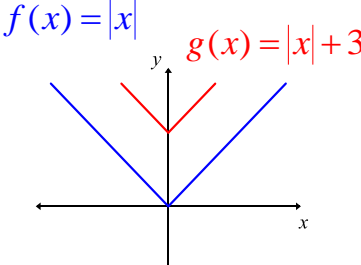
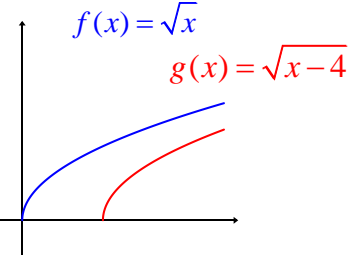
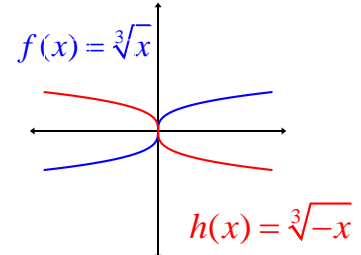
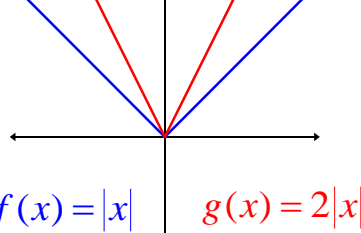
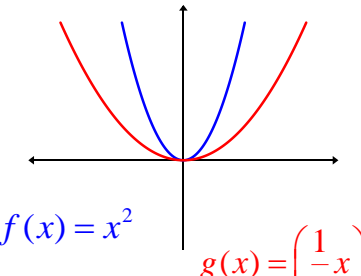
Example: Calculate the difference quotient for the function  $f(x) = 2x^2 - 7x - 11$

**Basic Functions**

<p>• Linear <math>f(x)=</math></p>  <p>Domain: Range:</p>	<p>• Quadratic <math>f(x)=</math></p>  <p>Domain: Range:</p>
<p>• Cubic <math>f(x)=</math></p>  <p>Domain: Range:</p>	<p>• Rational <math>f(x)=</math></p>  <p>Domain: Range:</p>
<p>• Radical <math>f(x)=</math></p>  <p>Domain: Range:</p>	<p>• Exponential <math>f(x)=</math></p>  <p>Domain: Range:</p>

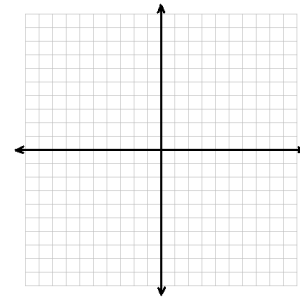


**Transformations of Functions**

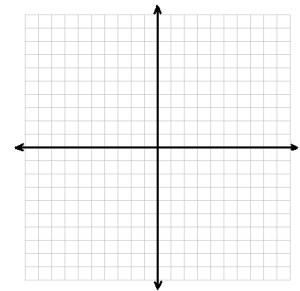
Transformation	Equation	Description	
<b>Vertical translation</b>	$y = f(x) + c$  $y = f(x) - c$	<hr/>  <hr/>	 <p><math>f(x) =  x </math> <math>g(x) =  x  + 3</math></p>
<b>Horizontal translation</b>	$y = f(x + c)$  $y = f(x - c)$	<hr/>  <hr/>	 <p><math>f(x) = \sqrt{x}</math> <math>g(x) = \sqrt{x - 4}</math></p>
<b>Reflections</b>	$y = -f(x)$  $y = f(-x)$	<hr/>  <hr/>	 <p><math>f(x) = \sqrt[3]{x}</math> <math>h(x) = \sqrt[3]{-x}</math></p>
<b>Vertical Stretching/Shrinking</b>	$y = cf(x)$	<hr/>  <hr/>	 <p><math>f(x) =  x </math> <math>g(x) = 2 x </math></p>
<b>Horizontal Stretching/Shrinking</b>	$y = f(cx)$	<hr/>  <hr/>	 <p><math>f(x) = x^2</math> <math>g(x) = \left(\frac{1}{2}x\right)^2</math></p>

Practice: Describe the change in the graph of the function  $f(x) = x^2$  for each of the following transformation, and then graph it.

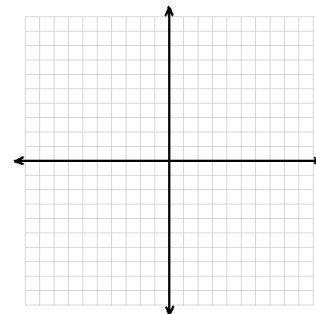
a.  $f(x) = x^2 + 2$  \_\_\_\_\_



b.  $f(x) = 2x^2$  \_\_\_\_\_

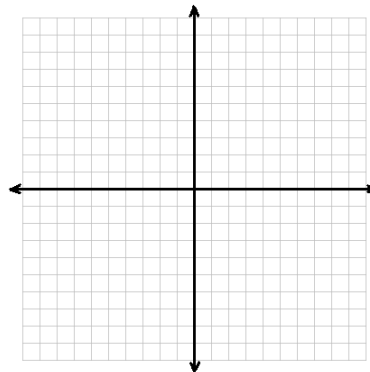
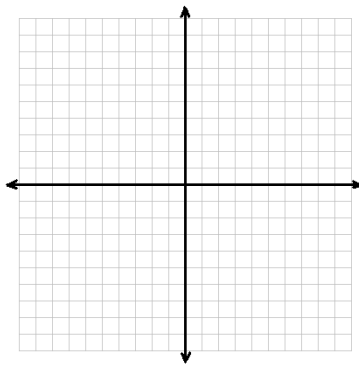


c.  $f(x) = -x^2$  \_\_\_\_\_



d.  $f(x) = (x+2)^2$  \_\_\_\_\_

e.  $f(x) = \left(\frac{1}{2}x\right)^2$  \_\_\_\_\_



## Class 6: Composition and Inverse Functions

### Objectives:

- ~ Combine functions using the algebra of functions
- ~ Determine domain of Functions and of composite functions
- ~ Write Functions as Compositions
- ~ Verify inverse functions
- ~ Find the Inverse of a Function
- ~ Determine if a function has an inverse
- ~ Graph a Function and its Inverses

### ➤ Domain of Functions

- **Domain:** \_\_\_\_\_

Practice: For each of the following functions, find the domain.

2.  $f(x) = 6x^4 - 3x^2 + 2x - 41$

3.  $h(x) = \frac{5 - 2x}{3x^2 - 19x + 6}$

4.  $k(t) = \sqrt{2t - 16}$

**Algebraic Operations with Functions**

Four algebraic operations that we do with polynomial functions are:

---

Practice: Perform the indicated operations for the following functions.

$$f(x) = x - 3,$$

$$g(x) = 2x^2 + 3x - 2,$$

$$h(x) = 2x^3 - 5x^2 + 6,$$

5.  $f(x) + h(x) =$

6.  $h(x) - g(x) =$

7.  $f(x) \cdot g(x) =$

8.  $\frac{g(x)}{f(x)} =$

**Composition of Functions**

- The Composition of the function \_\_\_\_\_
- 

**Practice:** Perform the indicated operations for the following functions.

$$f(x) = x + 4 \quad g(x) = x^2 - 3x - 4$$

9. Compose  $f(x) \circ g(x)$

10. Compose  $g(f(x))(1)$

## Inverses

- The Inverse of a function  $f$  \_\_\_\_\_

\_\_\_\_\_

Example: Determine if  $f(x) = 2x + 6$  and  $g(x) = \frac{x}{2} - 3$  are inverse functions

- **Finding Inverse Functions**

***Steps To find Inverse Functions***

1.

2.

3.

4.

5.

Example: Find the inverse of  $f(x) = 3x + 1$

- **Existence of Inverse Functions**

Does every function have an inverse? \_\_\_\_\_

How do we determine if a function has an inverse? \_\_\_\_\_

- **Algebraically:** \_\_\_\_\_

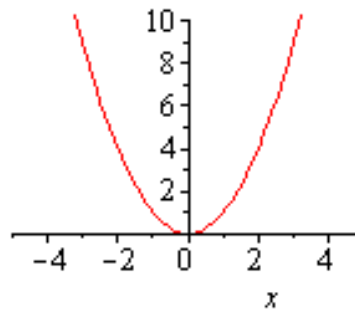
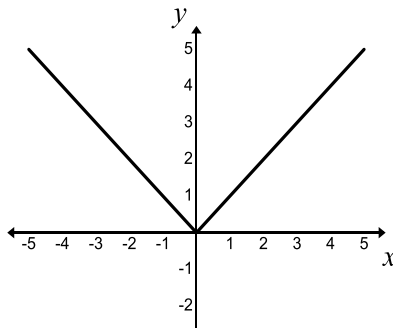
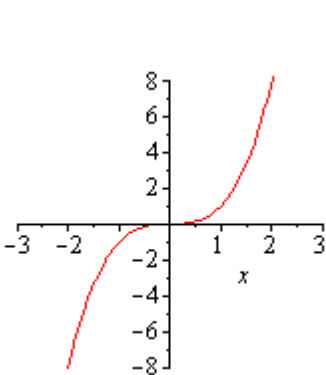
*Example:* \_\_\_\_\_

- **Graphically:** \_\_\_\_\_

**The horizontal Line Test:** \_\_\_\_\_

\_\_\_\_\_

Practice: Determine if the following functions have an inverse





## Class 7: Distance and Midpoint Formulas; Circles

### Objectives:

- ~ Find the Distance between two points.
- ~ Find the midpoint of a line segment
- ~ Write the standard form of a circle's equation
- ~ Give the center and radius of a circle whose equation is in standard form
- ~ Convert the general form of a circle's equation to standard form

• **The Distance Formula** \_\_\_\_\_

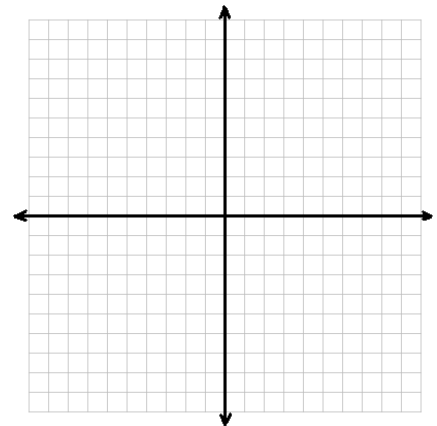
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• **The midpoint Formula** \_\_\_\_\_

---

1. Plot the points A(4, 6), B(-3, 2), and C (1,-5) on a coordinate system and connect them in order to find a triangle.

- a) Calculate the lengths of the three sides of the triangle.



## Circles

- A Circle is \_\_\_\_\_
- Radius is \_\_\_\_\_
- The Standard Equation of a circle is \_\_\_\_\_
- The General Form of the Equation of a Circle is \_\_\_\_\_

Practice: Write the standard equation for the circle in each of the following cases;

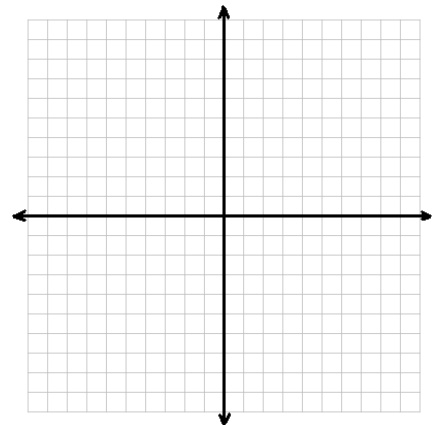
2. Center  $(-3, 5)$ ,  $r = 3$  \_\_\_\_\_

Practice: Give the center and radius of the circle described by the following equations:

3.  $(x+5)^2 + (y-4)^2 = 6$  \_\_\_\_\_

Practice: Complete the square and write the equation in standard form, then graph it and use it to identify the domain and range.

4.  $x^2 + y^2 + 8x + 4y + 16 = 0$



**Class 8 & 9 – Review and Test 1**

Summary/Questions

## Class 10: Angles and Their Measurements

### Objectives:

- ~ Define and draw angles
- ~ Convert angles from Degrees to Radians
- ~ Convert angles from Radians to Degrees
- ~ Use Right Triangles to Evaluate Trigonometric Functions

### ❖ Definitions:

Draw each of the following:

**Line:**

**Line Segment:**

**Ray :**

• **Angle:** \_\_\_\_\_

• **Standard Position:** \_\_\_\_\_

• **Positive Angles** \_\_\_\_\_

• **Negative Angles** \_\_\_\_\_

• **Quadrantal Angles** \_\_\_\_\_

- **Measuring Angles**

We measure angles by \_\_\_\_\_

- **By Degrees ( $^{\circ}$ )**

- **One Revolution** \_\_\_\_\_

We can classify angles by degrees:

- **Acute angle** \_\_\_\_\_

- **Right angle** \_\_\_\_\_

- **Obtuse Angle** \_\_\_\_\_

- **Straight angle** \_\_\_\_\_

*Practice:* Classify the following angles:

1.  $125^{\circ}$  -

2.  $160^{\circ}$  -

3.  $65^{\circ}$  -

4.  $90^{\circ}$  -

5.  $45^{\circ}$  -

6.  $180^{\circ}$  -

- **By Radians**
- **Central Angle:** \_\_\_\_\_
- **One Radian** \_\_\_\_\_
- **Radian Measure** \_\_\_\_\_

Example: Find the measure of the angle  $\theta$  that intercepts an arc of length 15 inches in a circle of radius 6 in.

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### Relationship between Degrees & Radians

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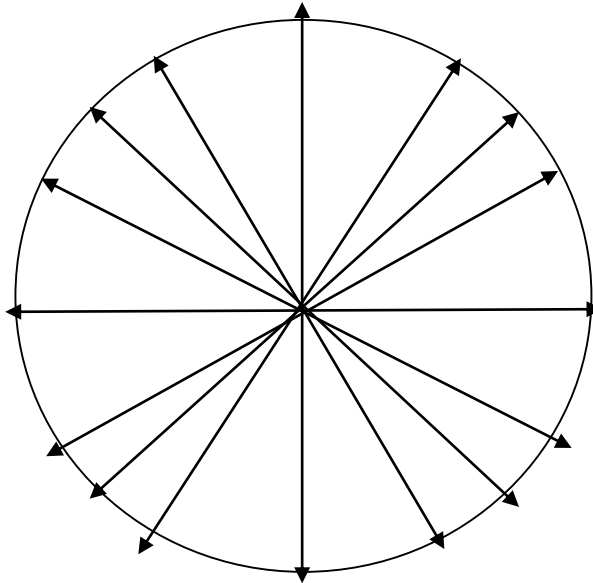
<b>Conversions:</b>
<ul style="list-style-type: none"><li>▪ To convert degrees to radians, _____</li><li>▪ To convert from radians to degrees, _____</li></ul>

Practice: Convert from radians to degree or degrees to radians as necessary

7.  $30^\circ =$

8.  $-\frac{5\pi}{3} \text{ radians} =$

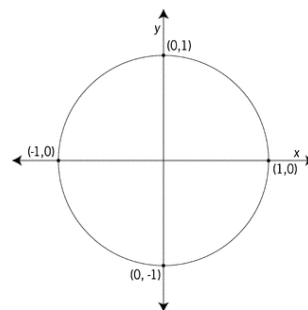
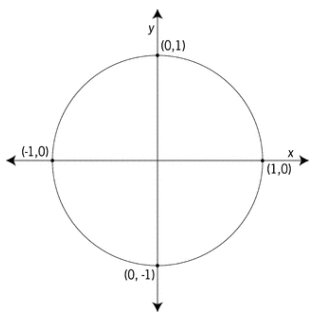
Fill the circle with the degree and radian measure



*Practice:* State the quadrant each angle is and then draw the angle in standard position.

9.  $\frac{3\pi}{5}$

10.  $-\frac{3\pi}{4}$



## Trigonometric Functions

- **Trigonometric Functions**

---

---

$$\sin \theta =$$

$$\csc \theta =$$

$$\cos \theta =$$

$$\sec \theta =$$

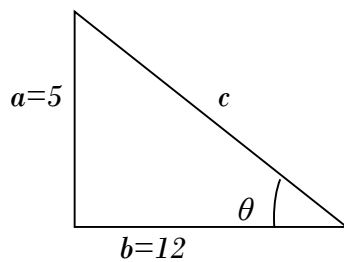
$$\tan \theta =$$

$$\cot \theta =$$

Do the values of the trigonometric functions depend on the length of the sides of a triangle?

---

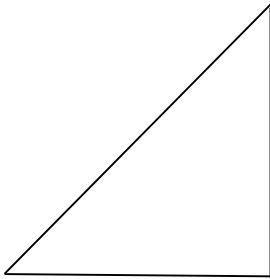
11: Find the value of each of the six trigonometric functions of  $\theta$  for the following triangle.





- Special Angles
- $30^\circ$ ,
- $45^\circ$ ,
- $60^\circ$
- or
- $\frac{\pi}{6}$ ,
- $\frac{\pi}{4}$ ,
- $\frac{\pi}{3}$

$45^\circ$  or  $\frac{\pi}{4}$  \_\_\_\_\_



$$\sin \frac{\pi}{4} =$$

$$\csc \frac{\pi}{4} =$$

$$\cos \frac{\pi}{4} =$$

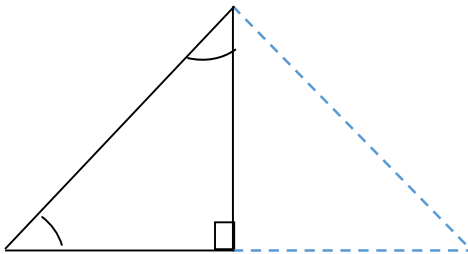
$$\sec \frac{\pi}{4} =$$

$$\tan \frac{\pi}{4} =$$

$$\cot \frac{\pi}{4} =$$

- $30^\circ$  or  $\frac{\pi}{6}$  and  $60^\circ$  or  $\frac{\pi}{3}$  \_\_\_\_\_

Example:



$$\sin \frac{\pi}{3} =$$

$$\sin \frac{\pi}{6} =$$

$$\cos \frac{\pi}{3} =$$

$$\cos \frac{\pi}{6} =$$

$$\tan \frac{\pi}{3} =$$

$$\tan \frac{\pi}{6} =$$

## Special Identities

***Reciprocal Identities:***

$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\sec \theta =$

$\cot \theta =$

- Pythagorean Identities:***

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---

*Example:* Given that  $\sin \theta = \frac{1}{2}$  and  $\theta$  is acute, find  $\cos \theta$

*Practice:* Use identities to find the trigonometric function.

12. Find  $\sin \theta$  if  $\cos \theta = \frac{7}{8}$

13. Find  $\tan \theta$  if  $\sin \theta = \frac{6}{7}$

**Co-terminal Angles****Co-terminal angles** \_\_\_\_\_  
\_\_\_\_\_

Example:

Practice: Find a positive angle less than  $2\pi$  that is co-terminal with each of the following.

14.  $400^\circ$

15.  $-135^\circ$

## Class 11: Trigonometric Functions of any Angle

### Objectives:

- Trigonometric functions of any angle/Definition
- Use the signs of the trigonometric functions
- Reference Angle
- Applications of Trigonometric Functions

### Definition of Trigonometric Functions of any Angle:

---

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---

Sin  $\theta =$

Csc  $\theta =$

Cos  $\theta =$

Sec  $\theta =$

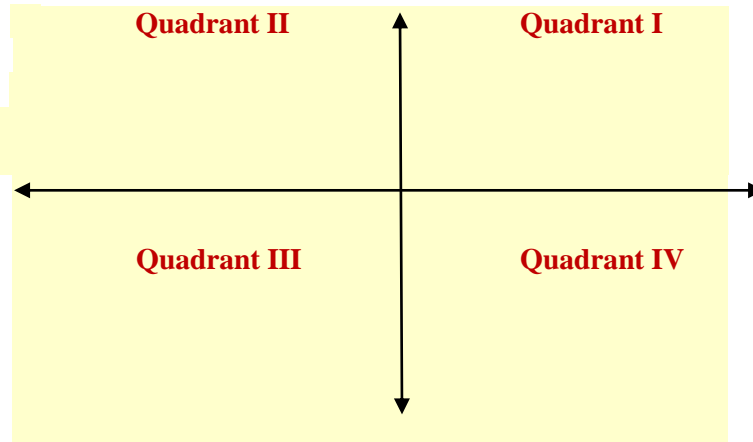
Tan  $\theta =$

Cot  $\theta =$

Example: Let  $P = (-3, -4)$  be a point in the terminal side of  $\theta$ . Find the value of the six trig. functions.

## The Signs of the Trigonometric Functions

The table summarizes the signs of the trigonometric functions

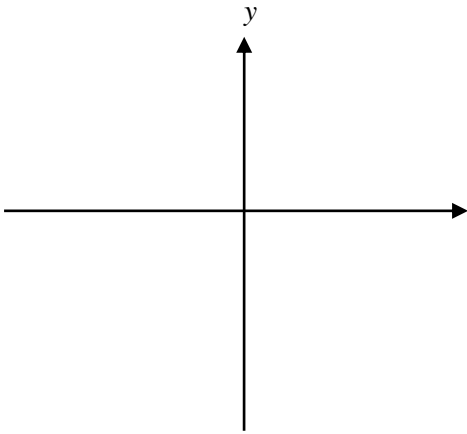


Here is an easy way to remember:

I	II	III	IV

Example: Given  $\tan \theta = -\frac{1}{3}$  and  $\cos \theta < 0$ , find  $\sin \theta$  and  $\sec \theta$ .

**Quadrantal Angles:** Lets find the values of trigonometric functions for the quadrantal angles.



$\theta$	$0$ $2\pi$	$90^\circ$ $\pi/2$	$180^\circ$ $\pi$	$270^\circ$ $3\pi/2$
$\sin\theta$				
$\cos\theta$				
$\tan\theta$				

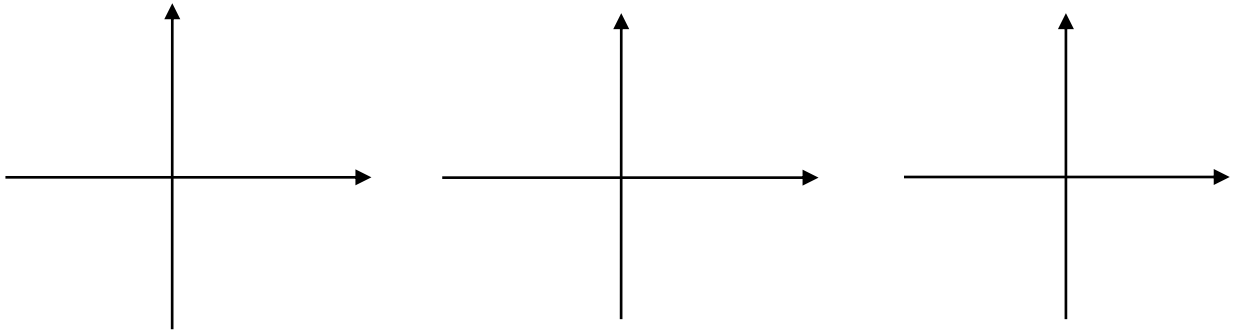
**Values of Special Angles**

$\theta$	$0^\circ$ $2\pi$	$30^\circ$ $\frac{\pi}{6}$	$45^\circ$ $\frac{\pi}{4}$	$60^\circ$ $\frac{\pi}{3}$	$90^\circ$ $\frac{\pi}{2}$	$180^\circ$ $\pi$	$270^\circ$ $\frac{3\pi}{2}$
$\sin \theta$							
$\cos \theta$							
$\tan \theta$							

## Reference Angles

- Reference Angles \_\_\_\_\_  
\_\_\_\_\_

Example:



### Finding Reference Angles:

If

\_\_\_\_\_

Example: Find the reference angle of

$$\theta = 210^\circ$$

$$\theta = \frac{7\pi}{4} \rightarrow \theta' =$$

Why do we need to know Reference Angles?

---

*Example:* Find the exact value of  $\cos \frac{4\pi}{3}$

*Practice:* Use identities to find the trigonometric function.

1. Find the exact value of  $\tan(-210^\circ)$

2. Find the exact value of  $\csc \frac{11\pi}{4}$



## Class 12: Trig Functions of Real Numbers & their Graphs

### Objectives:

- Trigonometric functions of real numbers
- Recognize Domain and Range of Sin and Cos functions
- Use of Even and Odd trigonometric Functions
- Use of Periodic Properties
- Graph the sine and cosine functions and their transformations

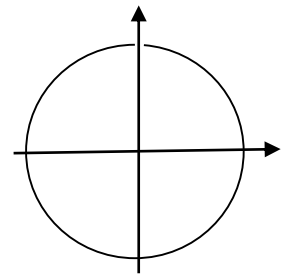
### ➤ Trigonometric Functions of Real Numbers

Cycles govern many aspects of our lives such as sleep patterns, seasons, tides etc. All follow regular, predictable cycles.

In this section we are going to see why trigonometric functions are used to model such phenomena.

*Until now we have considered trigonometric functions of angles. To define trigonometric functions of real numbers rather than angles we use a unit circle.*

**Unit Circle:** \_\_\_\_\_



### Definition of Trigonometric:

$$\sin \theta =$$

$$\cos \theta =$$

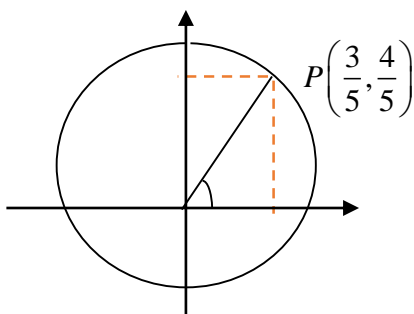
$$\tan \theta =$$

$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

Example:



## The Graph of Sine

$$y = \sin x$$

To graph a function \_\_\_\_\_

<b>x</b>	<b>0</b>	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	$2\pi$
<b>Y = sinx</b>													

Sketch a neat plot of the graph you got below:



### Characteristics of the basic function $y = \sin x$

**Domain:** \_\_\_\_\_

**Range:** \_\_\_\_\_

**Period:** \_\_\_\_\_

**Odd/Even:** \_\_\_\_\_

**x-intercepts:** \_\_\_\_\_

**Max/Min:** \_\_\_\_\_

**General Equation of Sine Function:** \_\_\_\_\_

**Amplitude** \_\_\_\_\_

**Period** \_\_\_\_\_

**Phase Shift** \_\_\_\_\_

**Vertical Shift** \_\_\_\_\_

Example: Determine the period, phase shift, and amplitude for  $y = 3 \sin\left(2x - \frac{\pi}{3}\right)$  and graph it.

## The Graph of Cosine

$$y = \cos x$$

We are going to graph  $y = \cos x$  also by listing some points on the graph.

To graph a function \_\_\_\_\_

<b>x</b>	<b>0</b>	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	$2\pi$
<b>Y = cosx</b>													

Sketch a neat plot of the graph you got below:



### Characteristics of the basic function $y = \cos x$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

Period: \_\_\_\_\_

Odd/Even: \_\_\_\_\_

x-intercepts: \_\_\_\_\_

Max/Min: \_\_\_\_\_

**General Equation of Cosine Function:** \_\_\_\_\_

Example: Determine the period, phase shift, and amplitude for  $y = 4 \cos(2x - \pi)$  and graph the function.

Practice: Graph the function  $y = -2 \cos(4\pi x + \pi) + 3$

## The Graph of Tangent

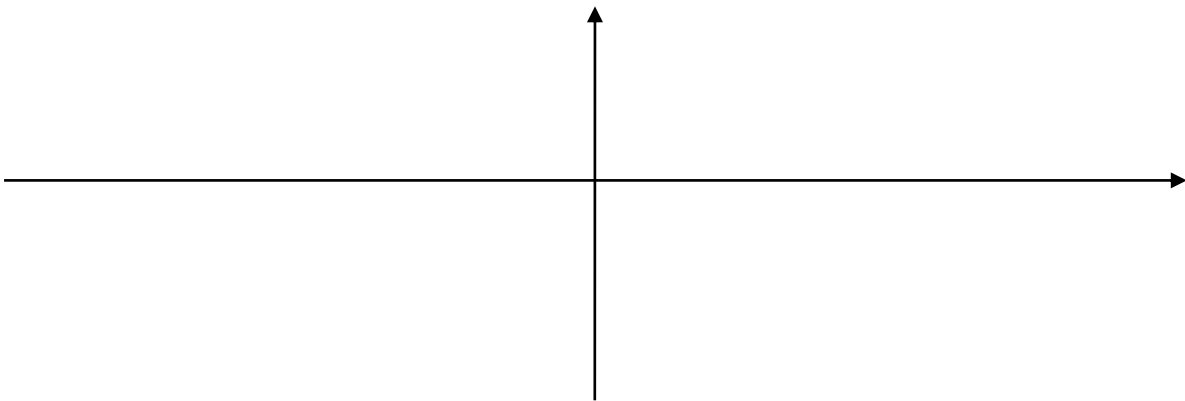
$$y = \tan x$$

We are going to graph  $y = \tan x$  also by listing some points on the graph.

To graph a function \_\_\_\_\_

<b>x</b>	<b>0</b>	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	$2\pi$
<b>Y = tanx</b>													

Sketch a neat plot of the graph you got below:



### Characteristics of the basic function $y = \tan x$

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

Period: \_\_\_\_\_

Odd/Even: \_\_\_\_\_

x-intercepts: \_\_\_\_\_

Max/Min: \_\_\_\_\_

**General Equation of Tangent Function:** \_\_\_\_\_

## Class 13: Inverse Trigonometric Functions & Applications

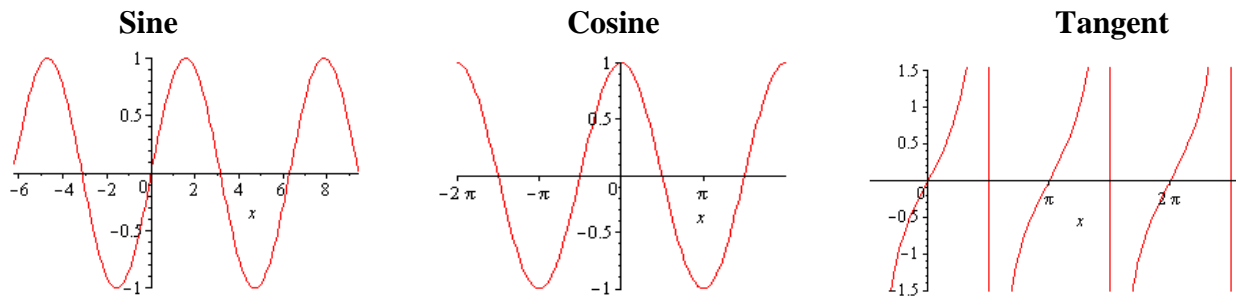
**Objectives:**

- ~ Understand and use the inverse Sine, Cosine and Functions
- ~ Use calculators to evaluate inverse trigonometric functions
- ~ Find exact values of composite functions with inverse trigonometric functions
- ~ Solve a Right Triangle
- ~ Application Of Trigonometric Functions

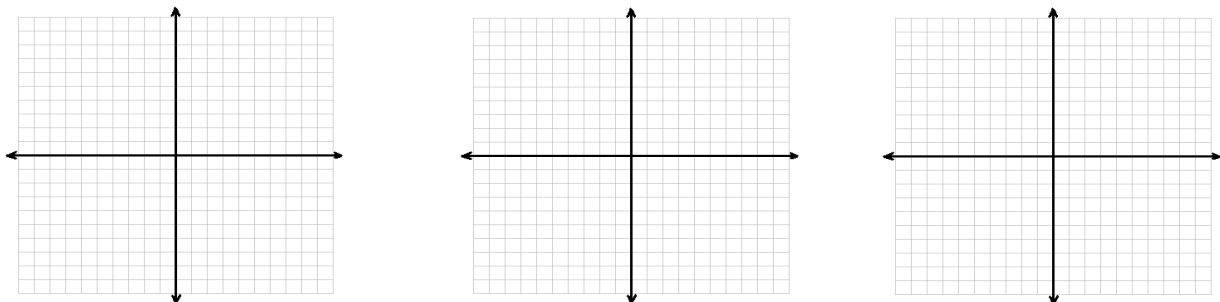
**RECALL:**

- 
- 
- 

The graph of the trigonometric functions are below:



If we restrict the domain of these functions we will get the following graphs:



Domain: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Range \_\_\_\_\_

\_\_\_\_\_

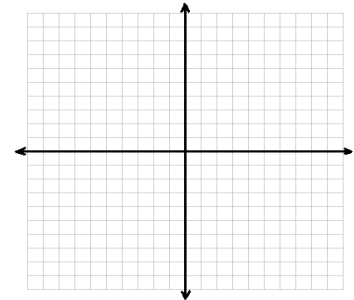
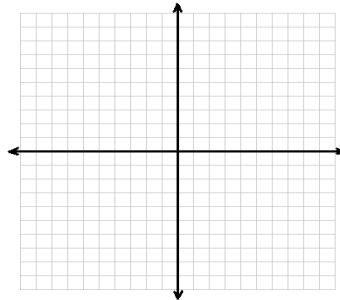
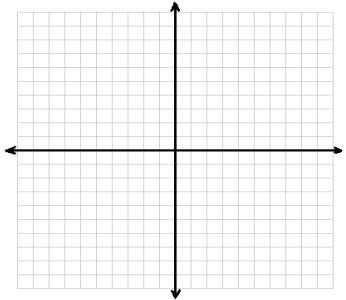
\_\_\_\_\_

The graph of the inverse trigonometric functions are below:

**Arcsine**

**Arcosine**

**Arctangent**



Function: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Domain: \_\_\_\_\_

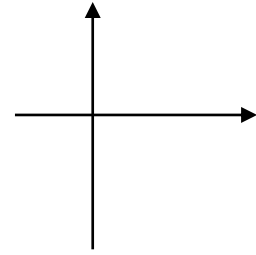
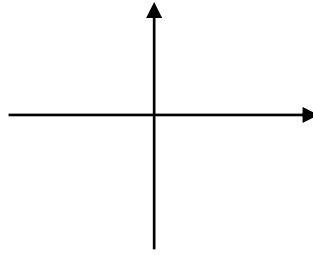
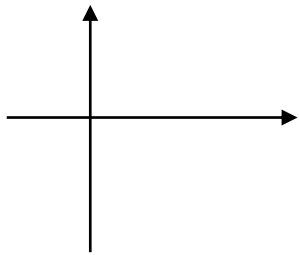
\_\_\_\_\_

\_\_\_\_\_

Range \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



Example: Find the exact value of each of the following:

$$\sin^{-1} \frac{\sqrt{2}}{2} =$$

$$\arccos \frac{\sqrt{3}}{2} =$$

$$\sin^{-1} 3 =$$

$$\cos^{-1}(-1) =$$

$$\tan^{-1}(0) =$$

$$\arctan\left(-\frac{\sqrt{3}}{3}\right) =$$



**Properties of Inverse Functions**

- $\left. \begin{array}{l} \sin(\sin^{-1} x) = \underline{\hspace{10em}} \\ \sin^{-1}(\sin x) = \underline{\hspace{10em}} \end{array} \right\}$

- $\left. \begin{array}{l} \cos(\cos^{-1} x) = \underline{\hspace{10em}} \\ \cos^{-1}(\cos x) = \underline{\hspace{10em}} \end{array} \right\}$

- $\left. \begin{array}{l} \tan(\tan^{-1} x) = \underline{\hspace{10em}} \\ \tan^{-1}(\tan x) = \underline{\hspace{10em}} \end{array} \right\}$

Using Inverse Properties:

Evaluate

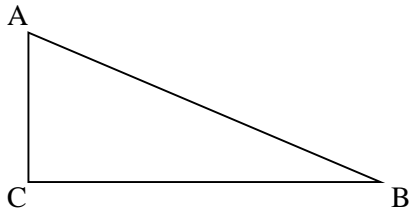
$$\sin^{-1}\left(\sin \frac{\pi}{4}\right) =$$

$$\tan(\tan^{-1}(-5)) =$$

$$\cos(\cos^{-1} \pi) =$$

**Solving Right Triangles**

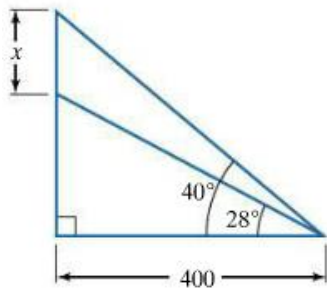
Solving a Triangle means \_\_\_\_\_



1. Let  $A = 62.7^\circ$  and  $a = 8.4$ . Solve the right triangle shown below rounding to two decimal place

Find  $x$  to the nearest whole unit

2.



## Angle of Elevation and Angle of Depression

- **Angle of Elevation** is \_\_\_\_\_

- **Angle of Depression** is \_\_\_\_\_

Example : From a point on a level ground 80 ft from the base of Eiffel Tower, the angle of elevation is  $85.4^\circ$ . Approximate the height of the Eiffel Tower to the nearest foot.

## Class 14 – Trigonometric Identities

### Objectives:

~ Use various methods to verify Trigonometric Identities

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

Example: Verify  $\csc x \tan x = \sec x$

## Class 15 – Trigonometric Equations

- Trigonometric Equation
- 

### Steps in Solving Trigonometric Equations

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

Practice: Solve the following equations:

1.  $5 \sin x = 3 \sin x + \sqrt{3}$

2.  $2\sin^2 x - 3\sin x + 1 = 0 \quad 0 \leq x \leq 2\pi$

3.  $\sin x \tan x = \sin x \quad 0 \leq x \leq 2\pi$

4.  $2\sin^2 x - 3\cos x = 0 \quad 0 \leq x \leq 2\pi$

## Class 16: The Law of Sines & The Law of Cosines

*Objectives:*

- Use the Law of Sines and Cosines to solve oblique triangles
- Solve applied problems using the Law of Sines and Cosines

- **Oblique Triangle** \_\_\_\_\_

**Note:** \_\_\_\_\_

- **The Law of Sines:** \_\_\_\_\_  
\_\_\_\_\_

Example: Solve the triangle ABC if  $A = 40^\circ$ ,  $C = 22.5^\circ$  and  $b=12$ .

Practice:

1. Solve the triangle ABC if  $A = 57^\circ$ ,  $a = 33$  and  $b = 26$ .

2. Solve the triangle ABC if  $A = 35^\circ$ ,  $a = 12$  and  $b = 16$ .



- The Law of Cosines \_\_\_\_\_

---

---

---

When given all three sides \_\_\_\_\_

---

Example: Solve the triangle with  $A=120^\circ$ ,  $b=7$  and  $c=8$ .

Example: Solve the triangle ABC if  $a = 8$ ,  $b = 10$  and  $c = 5$ .

**Class 17 & 18 – Review and Test 2**

Summary/Questions

## Class 19: Quadratic Functions

*Objectives:*

- Recognize Characteristics of Parabolas
- Graph Parabolas
- Determine a Quadratic Function's Max/Min Value
- Solve problems involving a quadratic function's max/min value.

- **Basic quadratic function** \_\_\_\_\_
- **Vertex** \_\_\_\_\_
- **Standard Form of a Quadratic Equation** \_\_\_\_\_
  - \_\_\_\_\_
  - \_\_\_\_\_
  - \_\_\_\_\_

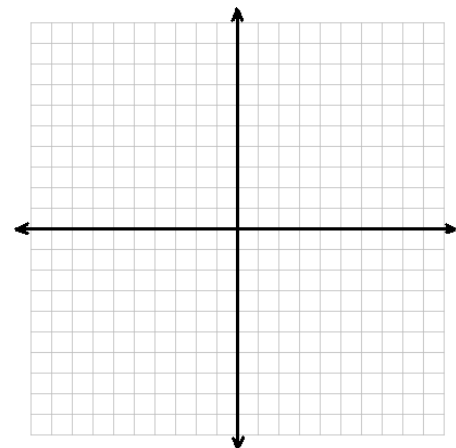
Practice: Identify the vertex and axis of symmetry of each parabola below.

1.  $f(x) = 3(x-2)^2 + 5$  \_\_\_\_\_

- To graph a quadratic Function in Standard form \_\_\_\_\_

Practice: Graph the following quadratic functions

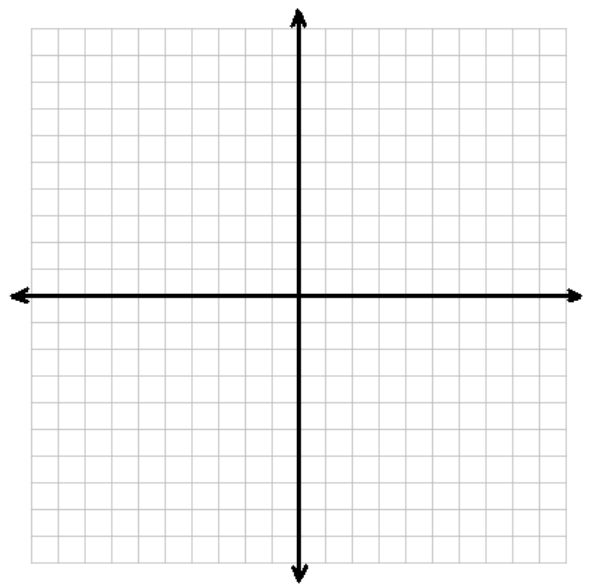
a.  $f(x) = 2(x-3)^2 + 1$



- **General Form of a Quadratic Equation** \_\_\_\_\_
- **Vertex** \_\_\_\_\_

Practice: Find the vertex for  $f(x) = x^2 + 3x - 10$

Practice: Graph the following quadratic functions  $f(x) = 2x^2 + 4x - 3$



## Class 20: Polynomial Functions & Division of Polynomials

*Objectives:*

- Identify polynomial functions
- Recognize characteristics of graphs of Polynomial Functions
- Determine end behavior
- Identify zeroes and their multiplicities
- Use synthetic division to divide polynomials
- Use the Rational Zero Theorem to find possible rational zeros
- Find zeros of Polynomial Functions

### • Polynomial Functions \_\_\_\_\_

Practice: Which of the following functions are polynomial functions?

a.  $f(x) = 3x^7 + 5x^3 + x^2 - 2$

b.  $f(x) = -2x^4 + 5x^{-2} + 8x$

c.  $f(x) = x^{2/3} + 7x^3 + \sqrt{x} - 2$  \_\_\_\_\_

### • Graphs of Polynomial Functions \_\_\_\_\_

### • Smooth \_\_\_\_\_

### • Continuous \_\_\_\_\_

### • End Behavior \_\_\_\_\_

• **If the degree is odd** \_\_\_\_\_

• **If the degree is even** \_\_\_\_\_

• **If the leading coefficient is positive** \_\_\_\_\_

• **If the leading coefficient is negative** \_\_\_\_\_

Example: Determine the end behavior of  $f(x) = 5x^3 + 7x^2 - x + 5$

---

Practice: Determine the end behavior of  $f(x) = -x^6 + 2x^5 + 7x^2 - 1$

---

• **Zeros of Polynomial Functions**

- \_\_\_\_\_

- \_\_\_\_\_

- \_\_\_\_\_

- \_\_\_\_\_

- \_\_\_\_\_

Example: Find the zeros of the polynomial functions below

$$f(x) = 2(x-3)^2(x+1)$$

---

$$f(x) = x^3 + 7x^2 - 4x - 28$$

---

**Synthetic Division**

2.  $(3x^2 - 5x + 5) \div (x - 5)$

3. 
$$\frac{(6x^5 - 2x^3 + 4x^2 - 5x + 5)}{x - 2}$$

Use synthetic division to evaluate  $f(1)$  for  $f(x) = 4x^3 - 12x^2 + 7x - 2$

## Zeros of Polynomial Functions

- **Rational Roots of Polynomial Functions**

---

Example: List all the possible rational zeroes of  $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

---

Practice: Find all possible rational zeros and use long/synthetic division to test them and find the actual ones for  $f(x) = x^3 + x^2 - 4x - 4$



## Class 21: Rational Functions

Objectives:

- Find the Domain of Rational Functions
- Identify Vertical Asymptotes
- Identify Horizontal Asymptotes
- Applications of Rational Functions

- **Rational Function** \_\_\_\_\_

Give at least two Examples \_\_\_\_\_

- **Domain** \_\_\_\_\_

- **To find the domain** \_\_\_\_\_  
\_\_\_\_\_

Example: Find the domain of  $f(x) = \frac{x^2 + 2x + 9}{(x-3)(x^2 - 16)}$

\_\_\_\_\_

- The Basic Rational Function is \_\_\_\_\_

Domain :

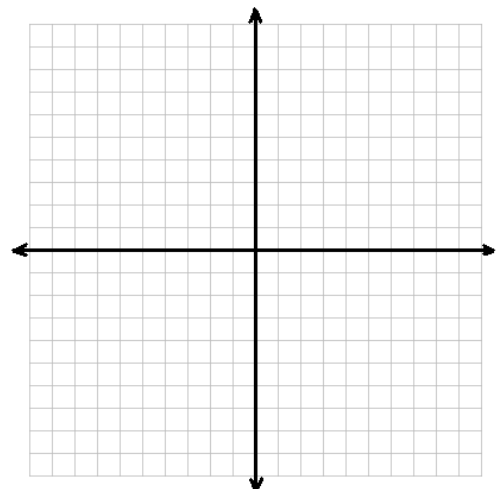
Range :

Y – int :

X – int :

Asymptotes :

Directional Limits :



## Asymptotes

- An Asymptote is \_\_\_\_\_  
\_\_\_\_\_
- **Finding Asymptotes:**
  - Vertical \_\_\_\_\_
  - Horizontal \_\_\_\_\_
  
  - Slant \_\_\_\_\_

*Practice:* Find all Asymptotes for each of the following functions

1.  $f(x) = \frac{x+2}{x^2+x-6}$

\_\_\_\_\_

2.  $f(x) = \frac{x^3-27}{x-3}$

\_\_\_\_\_

3.  $f(x) = \frac{2x^3+1}{3x^3+2x}$

\_\_\_\_\_

## Characteristics and Graphs of Rational Functions

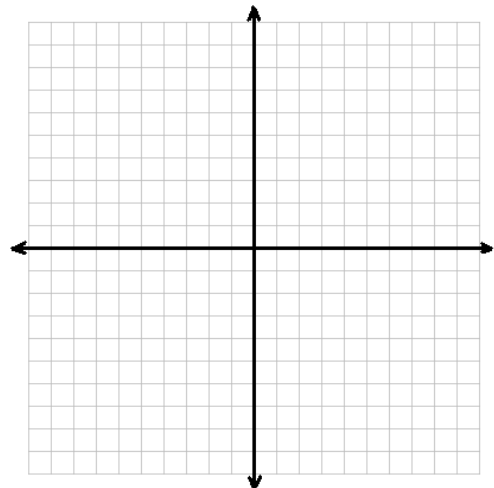
What do we need to know about R.F. ?

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

Example Give the characteristics and sketch a graph for each of

4.  $f(x) = \frac{2x+1}{x+1}$

- Domain:
  - Range:
  - x-int:
  - y-int:
  - HA:
  - VA:
  - SA:
- D. Limits:





## Class 22: Exponential and Logarithmic Functions

Review:

**Basic Laws of Exponents:** Write the exponential rules for each of the following (See pg. 4)

•  $a^x \cdot a^y = \underline{\hspace{2cm}}$        $(ab)^x = \underline{\hspace{2cm}}$        $a^1 = \underline{\hspace{2cm}}$

•  $\frac{a^x}{a^y} = \underline{\hspace{2cm}}$        $\left(\frac{a}{b}\right)^x = \underline{\hspace{2cm}}$        $a^0 = \underline{\hspace{2cm}}$

•  $(a^x)^y = \underline{\hspace{2cm}}$        $a^{-x} = \underline{\hspace{2cm}}$        $a^{x/y} = \underline{\hspace{2cm}}$

**Simplify the following:**

$$\frac{2^{27}}{2^{24}} =$$

$$(5)^{-3} =$$

$$(2^2)^3 =$$

$$(16)^{3/2} =$$

$$(2x^3y^4)^5 =$$

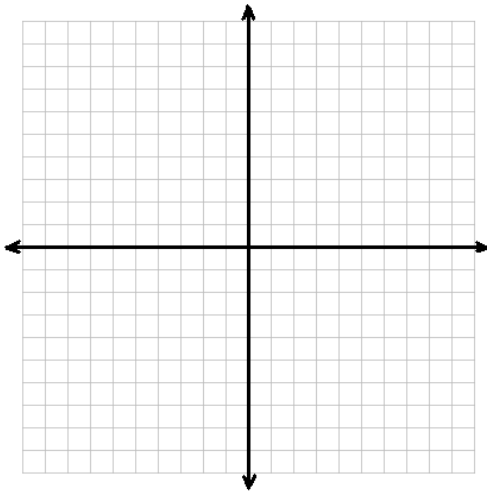
$$\left(\frac{1}{3}\right)^{-3} =$$

$$\sqrt[5]{(32)^3} =$$

$$(27)^{1/3} =$$

## Exponential Functions

- The exponential function \_\_\_\_\_



- y-intercept:** \_\_\_\_\_
- Domain:** \_\_\_\_\_
- Range:** \_\_\_\_\_
- Asymptotes?** \_\_\_\_\_
- Inverse** \_\_\_\_\_

In college, we study large volumes of information that, unfortunately we do not often retain for very long. The function  $f(x) = 80e^{-0.5x} + 20$  describes the percentage of information that a person can be expected to remember  $x$  weeks after learning it.

- Let  $x = 0$  and give the value of  $f(0)$

- Let  $x = 52$  and determine the value of  $f(52)$  accurate to the nearest ten thousandth

## Compounding

• **Simple Interest** \_\_\_\_\_

• **Compound Interest** \_\_\_\_\_

---

• **The Natural Base  $e$**  : \_\_\_\_\_

• **Continuous Compounding:** \_\_\_\_\_

Laura borrows \$2500 at a rate of 10.5%. Find how much Laura owes at the end of 4 years if:

- The interest is compounded yearly
- The interest is compounded quarterly
- The interest is compounded monthly
- The interest is compounded continuously
- Which option would yield the most interest, 10.5% compounded monthly for 4 years or 9% compounded continuously?

## Logarithmic Functions

- **The Logarithmic Function** \_\_\_\_\_

---

**A logarithm as an exponent:** \_\_\_\_\_

Write the Basic Laws of Logarithms below:

\_\_\_\_\_

\_\_\_\_\_

Write the following in its equivalent exponential form:

1.  $4 = \log_2 16$  \_\_\_\_\_

2.  $6 = \log_2 64$  \_\_\_\_\_

3.  $\log_6 216 = y$  \_\_\_\_\_

4.  $\log_5 125 = y$  \_\_\_\_\_

Write the following in its equivalent logarithmic form:

5.  $\sqrt[3]{8} = 2$  \_\_\_\_\_

6.  $13^2 = x$  \_\_\_\_\_

7.  $5^{-3} = \frac{1}{125}$  \_\_\_\_\_

8.  $7^y = 200$  \_\_\_\_\_



## Natural Logarithm

- The Natural Logarithmic Function \_\_\_\_\_

- **Properties of  $\ln(x)$ .**

Write the properties of the natural logarithm below:

_____	_____
_____	_____

**Simplify the following**

1.  $\ln e^6$

2.  $\log 10^7$

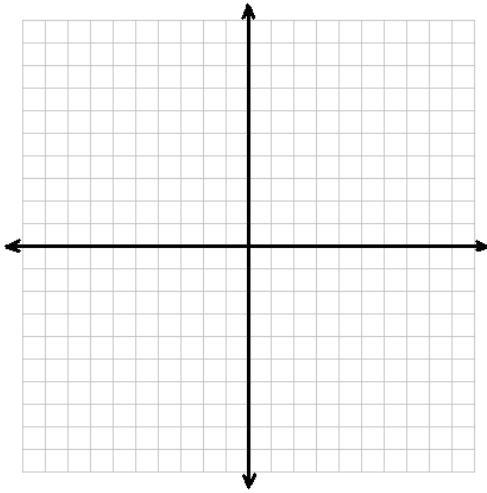
3.  $\ln \frac{1}{e^6}$

4.  $10^{\log 53}$

5.  $e^{\ln 125}$

6.  $e^{\ln 7x^2}$

## Logarithmic Functions



- **y-intercept:** \_\_\_\_\_
- **Domain:** \_\_\_\_\_
- **Range:** \_\_\_\_\_
- **Asymptotes?** \_\_\_\_\_
- **Inverse** \_\_\_\_\_

Example: Find the domain of the following functions:

a.  $f(x) = \ln(x - 2)$

b.  $f(x) = \log(3x + 6)$

\_\_\_\_\_

\_\_\_\_\_

The percentage of adult height attained by a girl who is  $x$  years old can be model  $f(x) = 62 + 35 \log(x - 4)$  where  $x$  represents the girl's age and  $f(x)$  represents the percentage of her adult height.

a. Approximately what percentage of her adult height has a girl attained at age 13?

b. Approximately what percentage of her adult height has a girl attained at age 16?

## Class 23: Exponential and Logarithmic Equations and Logistic Growth

- Exponential Equation

---

Give at least two example: \_\_\_\_\_

Examples: Solve the following equations

1.  $4^{2x-1} = 64$

2.  $3^{x-1} = \frac{1}{27}$

3.  $e^{x+4} = \frac{1}{e^{2x}}$

**Steps in Solving Exponential Equations**

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

*Example:* Solve for x  $3e^{5x} - 2 = 1977$

4.  $5e^{0.002x} - 9 = 12$

## Logistic Growth

- Logistic Model \_\_\_\_\_  
\_\_\_\_\_

7. The logistic growth function  $f(t) = \frac{100,000}{1 + 500e^{-t}}$  describes the number of people,  $f(t)$ , who have become ill with influenza  $t$  weeks after its initial outbreak in a particular community.

- a. How many people became ill with the flu when the epidemic began?

- b. How many people were ill by the end of the fourth week?

- c. What is the limiting size of the population that becomes ill?

**Properties of Logarithms**

- **Product Rule:** \_\_\_\_\_

*Example:*  $\log[(x+2)(x+3)] = \log(x+2) + \log(x+3)$

- **Quotient Rule:** \_\_\_\_\_

*Example:*  $\log\left[\frac{(2x+3)}{(x-5)}\right] = \log(2x+3) - \log(x-5)$

- **Power Rule:** \_\_\_\_\_

*Example:*  $7\log(x-3) = \log(x-3)^7$

**Class 24 & 25 – Review and Test 3****Summary/Questions**

